

# Risk Attribution

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# Agenda

- Risk
  - Definition
  - Ex-Post vs Ex-Ante
  - Quantification
- Risk Attribution
  - Absolute : Contribution to the total volatility
  - Relative : Contribution to the Tracking Error

# Risk

- Risk is an exposure to uncertainty
- Focus on the uncertainty due to market prices
  - Market risk
- Impact of market risk
  - Absolute : negative returns
  - Relative : negative excess returns
- Quantification
  - Absolute : volatility, Value at Risk, Earnings at Risk, etc.
  - Relative : tracking error, systematic (betas) or specific risk, etc.

# Ex-Post

- Ex-post = Management control
  - Understand the past to
    - **control** (example, compliance)
    - identify the **strengths** and the **weaknesses** of the process (example, performance attribution)
  - Set up actions to **enhance** the portfolio management process
- Some principles
  - Quantitative **and** qualitative approaches give a better understanding
  - Quantitative models must be accepted and understood by all the third parties involved (managers, performance measurers and clients)
  - Quantification is not an « *art* », it must add value

# Ex-ante

- Ex-ante = Risk management
  - Understand the impact of external shocks on value and P&L
  - Understand the impact of management decisions on risk
- Some principles
  - Future is unknown. Modelling the future requires assumptions.
  - Modelling introduces a new risk: the model risk.
  - Sophistication through modelling add value. Management needs to understand the assumptions and the limits of the models.
  - Stress tests are usefull to explore the impacts beyond the model

# Ex-Post vs Ex-Ante

- Past is known, there are no more uncertainties. Past is risk free → **control and learn**
- Future is uncertain. What are the sources of uncertainties (risk)? How to model these sources of risk? (probability, simulation, extreme events) → **identify, measure and manage risk**
- Understanding the past is valuable for the future → **ex-post risk attribution add value**



**We focus on ex-post absolute and relative risk attribution**

# Ex-post : Quantification

- Statistical analysis of time series data
  - Absolute: time series of returns
  - Relative: comparison of 2 or more time series; portfolio vs benchmark
- Standard statistical tools
  - Absolute: standard deviation, min, succes ratio, recovery period, etc.
  - Relative: tracking error, linear regression (betas, residuals), etc.

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# Ex-post : Risk attribution

- Question: what are the contribution to the risk of historical holdings and transactions?
  - Absolute : focus on the contribution to the total volatility of a sequence of holdings (transactions) securities.
  - Relative : focus on the contribution to the tracking error of a sequence of over(under)weights in securities

# Absolute Contribution

- **Contribution** = increase (decrease) of the total volatility due to a 1% change in holdings

$$C_{i,t}^p(\sigma) = \frac{\partial \sigma_p}{\partial w_{i,t}} = \frac{\partial}{\partial w_{i,t}} \left( \sum_{i,j}^N w_{i,t}^p \times w_{j,t}^p \times \sigma_{ij} \right) / 2\sigma_{p,t}$$

$$= \sum_j^N w_{j,t}^p \times \frac{\sigma_{ij}}{\sigma_{p,t}} = \rho_{ip,t} \times \sigma_i$$

- **Total Contribution** = Contribution times current holdings (in a security, a sector...)

$$TC_{i,t}^p(\sigma) = w_{i,t}^p \times \frac{\partial \sigma_p}{\partial w_{i,t}}$$

- **Total volatility** = sum of Total Contribution

$$\sigma_{p,t} = \sum_i^N w_{i,t}^p \times \frac{\partial \sigma_p}{\partial w_{i,t}}$$

# Comments

- The covariance matrix,  $\sigma_{ij}$  is required to calculate a contribution
- Problem of dimension, 250 holdings require more than 30.000 estimations.
  - A first solution is to reduce the dimension of the problem (factor analysis, PCA...)
  - A second solution, is to work directly on the portfolio's time series of returns

➡ **Focus on time series analysis**

# Contribution to volatility

- Total Contribution of a security over a period T is the sum of Total Contributions of each single period

$$TC_i(\sigma_p) = \rho_{p,CR_i} \times \sigma(CR_i)$$

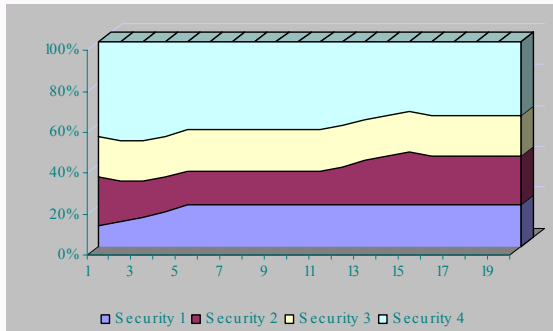
- $CR_i$  is the contribution of security i to the total return.  $\rho_{p,CR_i}$  is the correlation between portfolio returns and contribution to the return
- Required data
  - Historical returns are available at the security level
  - Returns are calculated on a given frequency (day, week, month). MWR over a single period include all transactions.

# Comments

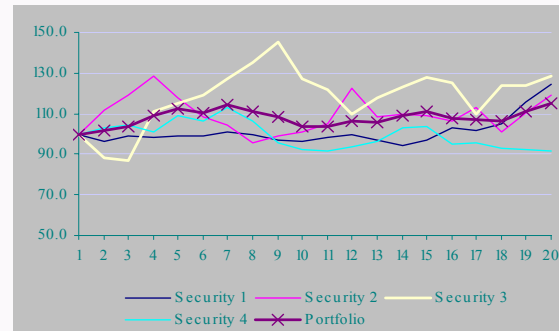
- The result is fundamental to risk attribution
  - Total Contribution takes into account historical transactions. The model allows weights to change through time
  - Calculation are based on available data
    - Historical returns of securities that are in the portfolio,
    - Historical holdings
  - The entire matrix variances-covariances is not required
  - Volatility of the return contribution explains the total risk. High security volatility + frequent trading = large contribution to the risk

# Example

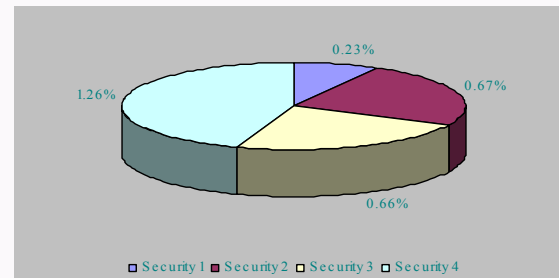
## Portfolio holdings



## Historical returns



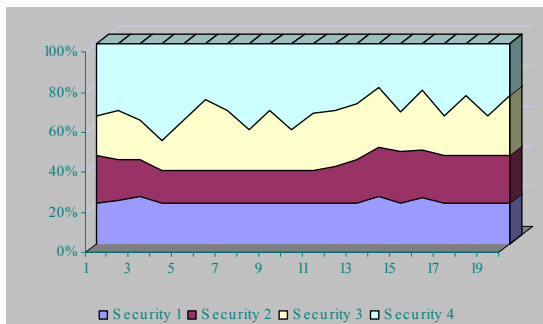
	Volatility	Correl	Contrib	Total Contrib
Security 1	3.72%	0.47	1.14%	0.23%
Security 2	8.18%	0.47	3.18%	0.67%
Security 3	9.74%	0.52	3.31%	0.66%
Security 4	4.84%	0.21	3.10%	1.26%
<b>Portfolio</b>	<b>2.824%</b>			<b>2.824%</b>



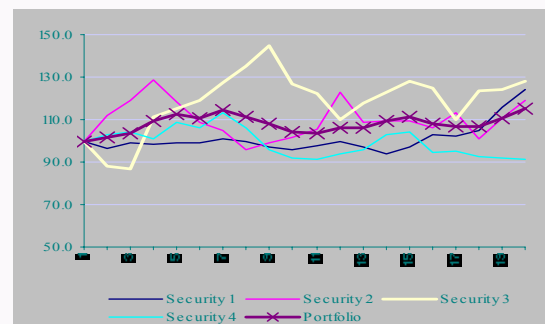
Security 3 has a the largest contribution to risk. Larger holdings and trading will increase the total contribution of security 3.

# Example

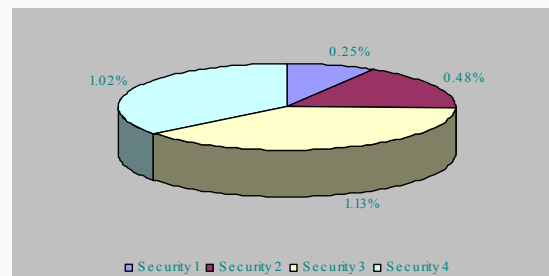
## Portfolio holdings



## Historical returns



	Volatility	Correl	Contrib	Total Contrib
Security 1	3.72%	0.47	1.20%	0.25%
Security 2	8.18%	0.47	2.21%	0.48%
Security 3	9.74%	0.52	4.52%	1.13%
Security 4	4.84%	0.21	2.83%	1.02%
<b>Portfolio</b>	<b>2.878%</b>			<b>2.878%</b>



Large holding and trading in security 3 have increased the total contribution

# Ex-Ante

- Contribution = correlation between a security and the existing portfolio times the security expected volatility
- Total Contribution = actual weight times Contribution



**To calculate the ex-ante contribution, we need an estimation of the expected correlations and volatilities**

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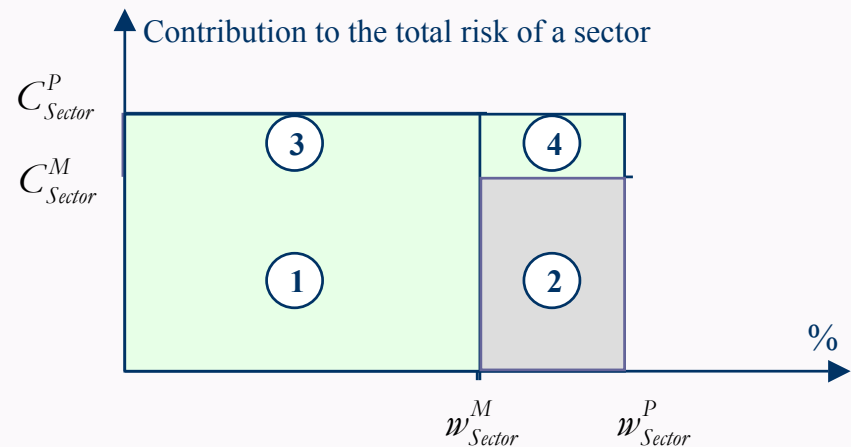
# Relative risk attribution

- 2 possible approaches
  - Comparison with a benchmark
  - Contribution to the Tracking Error

# Comparison with a benchmark

- Total volatility of the portfolio (benchmark) is a weighted sum of contributions

- 1 = contribution to the benchmark volatility
- 2 = contribution of over(under)-weighting a sector
- 3 = contribution of selecting different securities in the portfolio
- 4 = interaction effect



- The difference of volatility is

$$\sigma_P - \sigma_M = \sum_{Sector} (w_{Sector}^P - w_{Sector}^M) \times C_{Sector}^M + \sum_{Sector} w_{Sector}^P \times (C_{Sector}^P - C_{Sector}^M)$$

- Example 

# Comments

- The model explains the difference of volatility by allocation and selection
- Allocation is a part of the difference that is due to over(under)weighting a segment and investing in the index associated to it
- Selection is the remaining part that is due to selecting different securities



**This model is technically correct, but does it reflect the management process to control the volatility against the benchmark?**

**We will focus on tracking error attribution**

# Contribution to the TE

- What are the over(under)weights that have contributed to the actual level of TE?
- Contribution measures the impact on the TE of an over(under)weight

$$C_{i,t}^p(TE) = \frac{\partial TE}{\partial \Delta w_{i,t}}$$

- Total contribution of a security is the sum over T periods of a sequence of over(under)weights

$$C_{i,T}^p(TE) = \sum_{s=1}^T \Delta w_{i,s} \frac{\partial TE}{\partial \Delta w_{i,s}} = \rho(R_P - R_M; w_i^P R_{i,P} - w_i^M R_{i,M}) \times \sigma(w_i^P R_{i,P} - w_i^M R_{i,M})$$

- TE is equal to the sum of Total contributions

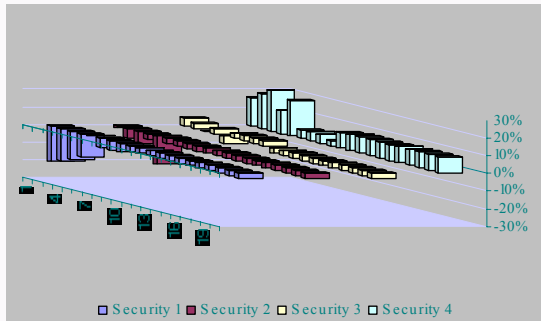
$$TE = \sum_{i=1}^N C_{i,T}^p(TE)$$

# Comments

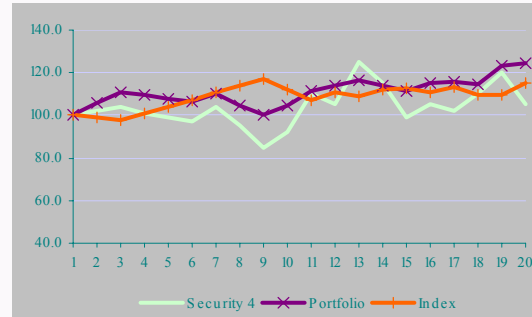
- TE contribution gives useful information of how the total TE have been generated. What holdings(transactions) in a particular security, asset class, sector... explain the historical level of TE
- Contribution to the TE depends on the
  - Correlation between the portfolio excess return and the difference of return contribution of the security (asset class, sector...)
  - Volatility of the difference of return contribution
- Calculation requires historical returns and contributions that are generally available
- The model supports time varying weights
- The model can be used for risk management (ex-ante)

# Example

Over(under)weights



Historical returns



	Volatility	Correl	Contrib
Security 1	3.72%	0.49	0.06%
Security 2	8.18%	0.60	0.17%
Security 3	5.32%	0.45	0.09%
Security 4	9.68%	0.49	0.74%
Portfolio	3.699%		1.0626%
Benchmark	3.334%		

Security 4 volatility is high and correlation is low. Overweighting security 4 has contributed to 70% of the historical tracking error

# Ex-ante

- For mandates that has TE ceiling, measuring the impact on TE of a proposed trade is necessary.
- The contribution gives the impact of a 1% over(under)weight
- It can be shown that the matrix of variances-covariances is required to calculate the contribution

# Conclusions

- Risk attribution is a management control tool
  - Control the management process
  - Understandable and accepted by all third parties
  - By retroaction, enhance the management process
- Risk attribution is a risk management tool
  - Understand the impact of actual decisions
  - Modelling rely on assumptions → Model risk